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THE STRANGENESS RADIUS AND MAGNETIC MOMENT OF THE NUCLEON REVISITED ^{#1}

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Abstract

We update Jaffe's estimate of the strange isoscalar radius and magnetic moment of the nucleon. We make use of a recent dispersion-theoretical fit to the nucleon electromagnetic form factors and an improved description of symmetry breaking in the vector nonet. We find $\mu_s = -0.24 \pm 0.03$ n.m. and $r_s^2 = 0.21 \pm 0.03$ fm². The strange formfactor $F_2^s(t)$ follows a dipole with a cut-off mass of 1.46 GeV, $F_2^s(t) = \mu_s(1 - t/2.14 \text{ GeV}^2)^{-2}$. These numbers should be considered as upper limits on the strange vector current matrix-elements in the nucleon.

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1 Introduction

Over the last years, there has been considerable activity to pin down strange matrix-elements in the nucleon. Much interest has been focused on the strange quark content to the proton spin as measured in DILS and the strange quark contribution to the nucleon mass as revealed in the analysis of the pion–nucleon σ –term. Jaffe [1] estimated the matrix elements of the operators $r_s^2 \equiv s^\dagger(\vec{x})\vec{x}^2 s(\vec{x})$ and $\mu_s \equiv \vec{x} \times \vec{s} \vec{\gamma} s$ using the dispersion theory fits to the nucleon isoscalar form factors of Höhler et al. [2]. Such signals of strangeness in the nucleon have also been considered in a variety of hadron models and have lead to dedicated experiments like SAMPLE at MIT-Bates, a flurry of CEBAF proposals, one proposal at MAMI and many other experimental as well as theoretical activities (for a review, see Ref.[3]). In this letter, we want to update the estimate of the strange magnetic moment and radius by incorporating various new developments not available at the time Ref.[1] was written. First, a new dispersion theoretical analysis of the nucleon form factors has been performed [4]. It improves upon the work of Höhler et al. [2] in various respects. These are the implementation of the constraints from perturbative QCD (pQCD) at large momentum transfer, the inclusion of the recent neutron–atom scattering length determination [5] to constrain the neutron charge radius and, of course, the inclusion of new data at low, moderate and high momentum transfer (as listed in [4]). In that paper strong support for the basic assumption of Jaffe’s analysis, namely the identification of the second pole in the isoscalar Dirac and Pauli formfactors with the $\phi(1020)$, is presented. In fact, even the location of the third pole necessary in the isoscalar channel could be identified with the mass of the $\omega(1600)$ (denoted by S' in [4]). Furthermore, the symmetry breaking of the strong and electroweak interactions in the vector nonet has been considerably refined [6] leading to an improved value of the $\omega\phi$ mixing angle ϵ . These are the ingredients we will use to update and sharpen the analysis of Jaffe.

2 Formalism

It is straightforward to generalize Jaffe’s parametrization of the isoscalar formfactors to account for the constraints from pQCD. We follow Ref.[4] and separate the spectral functions of the pertinent form factors into a hadronic (meson pole) and a quark (pQCD) component as follows,

$$F_i^{(I=0)}(t) = \tilde{F}_i^{(I=0)}(t)L(t) = \left[\sum_{I=0} \frac{a_I^{(I=0)} L^{-1}(M_{(I=0)}^2)}{M_{(I=0)}^2 - t} \right] \left[\ln \left(\frac{\Lambda^2 - t}{Q_0^2} \right) \right]^{-\gamma} \quad (1)$$

with

$$L(t) = \left[\ln \left(\frac{\Lambda^2 - t}{Q_0^2} \right) \right]^{-\gamma} . \quad (2)$$

Here, $\Lambda \simeq 10 \text{ GeV}^2$ separates the hadronic from the quark contributions, Q_0 is related to Λ_{QCD} and γ is the anomalous dimension,

$$F_i(t) \rightarrow (-t)^{-(i+1)} \left[\ln \left(\frac{-t}{Q_0^2} \right) \right]^{-\gamma}, \quad \gamma = 2 + \frac{4}{3\beta}, \quad i = 1, 2, \quad (3)$$

with β the QCD β -function and t the invariant momentum transfer squared. For the best fit to the available proton *and* neutron data, the isoscalar masses are $M_\omega = 0.782 \text{ GeV}$, $M_\phi = 1.019 \text{ GeV}$ and $M_{S'} = 1.60 \text{ GeV}$ with the corresponding residua being $a_1^\omega = 0.747$, $a_2^\omega = -0.122$, $a_1^\phi = -0.738$, $a_2^\phi = 0.162$, $a_1^{S'} = -0.0382$ and $a_2^{S'} = -0.0406$. The QCD parameters are $\gamma = 2.148$, $\Lambda^2 = 9.73 \text{ GeV}^2$ and $Q_0^2 = 0.35 \text{ GeV}^2$.

Apart from these changes, we adhere to the assumptions of Jaffe [1] concerning the definition of the $\omega\phi$ mixing angle ϵ ,

$$\begin{aligned} |\omega\rangle &= \cos \epsilon |\omega_0\rangle - \sin \epsilon |\phi_0\rangle \\ |\phi\rangle &= \sin \epsilon |\omega_0\rangle + \cos \epsilon |\phi_0\rangle \end{aligned} \quad (4)$$

as well as the parametrization of the on-mass-shell couplings of the pure vector states (ω_0, ϕ_0) to the nucleons,

$$g_i(\phi_0 NN) \equiv g_i \sin(\eta_i), \quad g_i(\omega_0 NN) \equiv g_i \cos(\eta_i), \quad (5)$$

with $i = 1, 2$ for the vector and the tensor coupling, respectively. We also ignore $\text{SU}(3)_f$ violations in the vector meson-current couplings.^{#5} This universal coupling strength of each quark q_k to the current $\bar{q}_k \gamma_\mu q_k$ is denoted by κ .

To be specific, consider first the (isoscalar) Dirac form factor $F_1(t)$,

$$\begin{aligned} F_1^{I=0}(t) = & \left[\frac{1}{2} L^{-1}(0) + \frac{t \kappa g_1}{\sqrt{6}} \left(\frac{\sin(\epsilon + \eta_1) \cos(\theta_0 + \epsilon)}{t - M_\phi^2} L^{-1}(M_\phi^2) \right. \right. \\ & \left. \left. - \frac{\cos(\epsilon + \eta_1) \sin(\theta_0 + \epsilon)}{t - M_\omega^2} L^{-1}(M_\omega^2) \right) - \frac{t A_1^{I=0}}{t - M_{S'}^2} L^{-1}(M_{S'}^2) \right] L(t) \end{aligned} \quad (6)$$

$$\begin{aligned} F_1^s(t) = & \left[-t \kappa g_1 \left(\frac{\sin(\epsilon + \eta_1) \cos \epsilon}{t - M_\phi^2} L^{-1}(M_\phi^2) \right. \right. \\ & \left. \left. - \frac{\cos(\epsilon + \eta_1) \sin \epsilon}{t - M_\omega^2} L^{-1}(M_\omega^2) \right) - \frac{t A_1^s}{t - M_{S'}^2} L^{-1}(M_{S'}^2) \right] L(t) \end{aligned} \quad (7)$$

with $\theta_0 = 35^\circ$ the ideal mixing angle. The following normalization conditions and constraints are fulfilled by construction,

$$F_1^{I=0}(t=0) = \frac{1}{2}, \quad F_1^s(t=0) = 0, \quad (8)$$

$$\lim_{t \rightarrow -\infty} F_1^{I=0}(t) = 0, \quad \lim_{t \rightarrow -\infty} F_1^s(t) = 0. \quad (9)$$

^{#5}This assumption should eventually be relaxed in a more refined analysis.

The constants $A_1^{I=0}$ and A_1^s are fixed by demanding

$$\lim_{t \rightarrow -\infty} L^{-1}(t) F_1^{I=0}(t) = 0, \quad \lim_{t \rightarrow -\infty} L^{-1}(t) F_1^s(t) = 0, \quad (10)$$

which leads to

$$A_1^{I=0} = L(M_{S'}^2) \left[\frac{1}{2} L^{-1}(0) + \frac{\kappa g_1}{\sqrt{6}} \left(\sin(\epsilon + \eta_1) \cos(\theta_0 + \epsilon) L^{-1}(M_\phi^2) - \cos(\epsilon + \eta_1) \sin(\theta_0 + \epsilon) L^{-1}(M_\omega^2) \right) \right] \quad (11)$$

$$A_1^s = L(M_{S'}^2) \left[-\kappa g_1 \left(\sin(\epsilon + \eta_1) \cos \epsilon L^{-1}(M_\phi^2) - \cos(\epsilon + \eta_1) \sin \epsilon L^{-1}(M_\omega^2) \right) \right] \quad (12)$$

and thus the formfactors $F_1^{I=0}(t)$ and $F_1^s(t)$ are determined.

In complete analogy, we parametrize the (isoscalar) Pauli formfactor $F_2(t)$ as

$$F_2^{I=0}(t) = \left[\frac{\kappa g_2}{\sqrt{6}} \left(\sin(\epsilon + \eta_2) \cos(\theta_0 + \epsilon) L^{-1}(M_\phi^2) \frac{M_\phi^2}{t - M_\phi^2} - \cos(\epsilon + \eta_2) \sin(\theta_0 + \epsilon) L^{-1}(M_\omega^2) \frac{M_\omega^2}{t - M_\omega^2} \right) - \frac{M_{S'}^2 A_2^{I=0}}{t - M_{S'}^2} L^{-1}(M_{S'}^2) \right] L(t) \quad (13)$$

$$F_2^s(t) = \left[-\kappa g_2 \left(\sin(\epsilon + \eta_2) \cos \epsilon L^{-1}(M_\phi^2) \frac{M_\phi^2}{t - M_\phi^2} - \cos(\epsilon + \eta_2) \sin \epsilon L^{-1}(M_\omega^2) \frac{M_\omega^2}{t - M_\omega^2} \right) - \frac{M_{S'}^2 A_2^s}{t - M_{S'}^2} L^{-1}(M_{S'}^2) \right] L(t) \quad (14)$$

subject to the constraints

$$\lim_{t \rightarrow -\infty} F_2^{I=0}(t) = 0, \quad \lim_{t \rightarrow -\infty} F_2^s(t) = 0, \quad (15)$$

$$\lim_{t \rightarrow -\infty} t F_2^{I=0}(t) = 0, \quad \lim_{t \rightarrow -\infty} t F_2^s(t) = 0 \quad (16)$$

Imposing furthermore

$$\lim_{t \rightarrow -\infty} L^{-1}(t) t F_2^{I=0}(t) = 0, \quad \lim_{t \rightarrow -\infty} L^{-1}(t) t F_2^s(t) = 0, \quad (17)$$

leads to

$$A_2^{I=0} = \frac{L(M_{S'}^2)}{M_{S'}^2} \frac{\kappa g_2}{\sqrt{6}} \left(\sin(\epsilon + \eta_2) \cos(\theta_0 + \epsilon) M_\phi^2 L^{-1}(M_\phi^2) - \cos(\epsilon + \eta_2) \sin(\theta_0 + \epsilon) M_\omega^2 L^{-1}(M_\omega^2) \right)$$

$$- \cos(\epsilon + \eta_2) \sin(\theta_0 + \epsilon) M_\omega^2 L^{-1}(M_\omega^2) \Big) \quad (18)$$

$$\begin{aligned} A_2^s = & -\frac{L(M_{S'}^2)}{M_{S'}^2} \kappa g_2 \left(\sin(\epsilon + \eta_2) \cos \epsilon M_\phi^2 L^{-1}(M_\phi^2) \right. \\ & \left. - \cos(\epsilon + \eta_2) \sin \epsilon M_\omega^2 L^{-1}(M_\omega^2) \right) \end{aligned} \quad (19)$$

and, consequently, $F_2^{I=0}(t)$ and $F_2^s(t)$ are given. However, as pointed out by Musolf [7], the asymptotic behaviour of the strange vector formfactors plays an important role in such type of analysis as presented here or by Jaffe [1]. Given the assumptions used in our analysis, the large momentum transfer behaviour of the strange formfactors $F_{1,2}^s(t)$ follows essentially the one of the isoscalar electromagnetic ones, $F_{1,2}^{I=0}(t)$. Quark counting rules suggest that the extra $\bar{s}s$ pair in the Fock-space decomposition of the nucleon wavefunction needed to describe $F_{1,2}^s(t)$ leads to a further t^2 -suppression as compared to the conventional isoscalar electromagnetic formfactors. Imposing such a constraint can lead to a significant reduction of the strange matrix-elements at low momentum transfer [7]. Since our assumption about the large- t fall-off of $F_{1,2}^s(t)$ can not be excluded at present, we consider the resulting numbers as upper limits. With this caveat in mind, we are now in the position to analyze the strange formfactors.

3 Results and discussion

First, we must fix parameters. In particular, there has been some dispute about the mixing angle ϵ . Jaffe used the value of $\epsilon = 0.053 \pm 0.005$ as determined in Ref.[8] from $\omega, \phi \rightarrow 3\pi$ decays. Since then, there have been some changes in certain decay modes which makes this determination to some extent uncertain. A more elaborate treatment of symmetry breaking has been proposed by Harada and Schechter [6]. They fit a wealth of data with a few parameters and in that scheme ϵ is determined from the decay mode $\phi \rightarrow \pi^0 \gamma$, $\epsilon = 0.052 \dots 0.056$. The central value used in [6] is $\epsilon = 0.055$. If one ignores the effect of $\pi^0 \eta$ mixing, then ϵ is reduced to 0.0325. These are the benchmark values we will use in the following.

In table 1, we show the numerical results of the fits to the nucleon isoscalar form factors of [4], for the central value of ϵ and the very small one as discussed before. The results are stable and within the uncertainty of Jaffe's calculation [1].

ϵ	κg_1	η_1	κg_2	η_2	r_2^s [fm ²]	μ_s [n.m.]
0.055	5.36	0.38	-0.93	0.50	0.21	-0.24
0.0325	5.48	0.39	-0.95	0.50	0.23	-0.25

Table 1: Parameters and strange matrix elements extracted from the dispersion-theoretical fit to the nucleon isoscalar formfactors of Ref.[4].

We stress that the fits of Ref.[4] exhibit a much smaller variation in the various parameters than it was the case in Ref.[2] due to the more tighter constraints. The largest uncertainty stems indeed from the value of ϵ . Adopting the procedure of Ref.[4] to estimate the uncertainties of the formfactor fits, we assign an error of $\pm 0.03 \text{ fm}^2$ to r_s^2 and of $\pm 0.03 \text{ n.m.}$ to μ_s . These, however, should be taken cum grano salis since we did not consider some other sources of uncertainty like e.g. $\text{SU}(3)_f$ violation in the vector meson–current couplings. These results are compatible with the rather uncertain determinations of $F_{1,2}^s(0)$ from νp elastic scattering data [9] (although in that paper, a negative value for r_1^s is preferred).^{#6} Various hadron models lead to a wide range of predictions, $\mu_s = -0.003 \dots -0.45 \text{ n.m.}$ and $r_s^2 = -0.25 \dots 0.22 \text{ fm}^2$ [11, 12, 13, 14, 15, 16, 17]. We remark that in the context of the Skyrme–type models, it is often stated that the large values of the resulting strange matrix–elements are an artefact of the $\text{SU}(3)$ symmetric wave functions. This deserves further study.

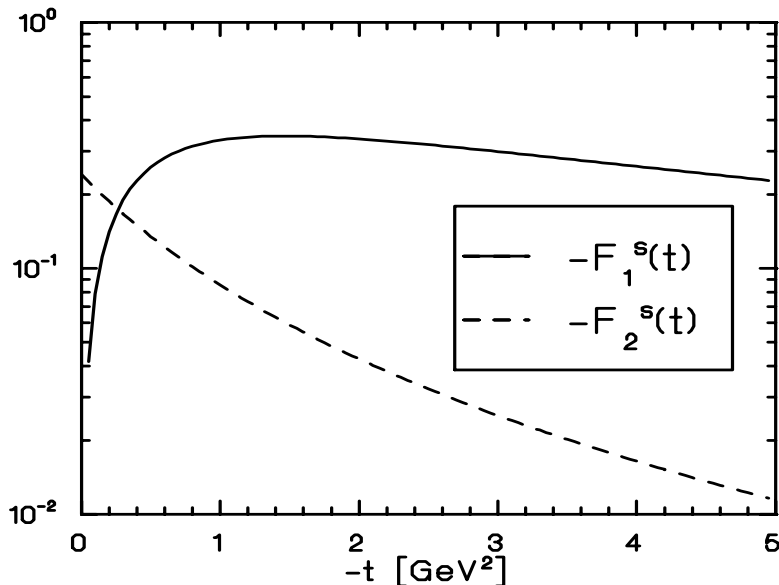


Fig. 1: The strange formfactors $F_{1,2}^s(t)$ for $\epsilon = 0.055$.

The strange formfactors $F_{1,2}^s(t)$ are shown in Fig. 1. We note that $F_1^s(t)$ reaches its maximum in the range of momentum transfer accessible to CEBAF. However, we stress

^{#6}That analysis, however, is based on the assumption of one unique cut–off mass M_A for all three axial–vector formfactors, $G^{(\alpha)}(t)$ ($\alpha = 0, 3, 8$). This assumption is at variance with expectations from hadron models which lead to a good description of the electroweak structure of the nucleon. For example, in Ref.[10] it was shown that $M_A^{(0)} = 1.2 M_A^{(3)}$ and the consequences for the extraction of strange matrix–elements were discussed.

again that this should be considered an upper limit since a faster large- t fall-off will certainly reduce the form factor [7]. The strange Pauli formfactor $F_2^s(t)$ can be fitted well by a dipole,

$$F_2^s(t) = \mu_s (1 - t/2.14 \text{ GeV}^2)^{-2} , \quad (20)$$

i.e. with a cut-off mass of 1.46 GeV.

To summarize, we have updated the analysis of Ref.[1] to deduce the strange form factors $F_{1,2}^s(t)$ from a dispersion-theoretical fit to the nucleons' isoscalar formfactors [4]. Both formfactors are negative and the strange radius and the magnetic moment are $r_1^2 = 0.21 \text{ fm}^2$ and $\mu_s = -0.24 \text{ n.m.}$, respectively. These numbers are to be considered as upper limits due to the large- t assumptions of $F_{1,2}^s(t)$ we made. We look forward to their experimental determinations.

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